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PRECALCULUS

POLYNOMIAL FUNCTION

Quadratic Functions

Graphing of Polynomial Functions

Polynomial Inequalities

IVT

Long Division

Factor and Remainder Theorem

Sections In the worksheet :

1. Practice Set (Skill Building)

2. Exam Set (Exam-style questions)

Note : For questions without space provided, solve on blank paper and attach to the worksheet.

Book No : 942-D

Section 1

Name : ………………………………..

School : ………………………………..

1

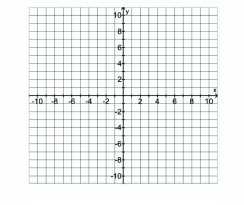
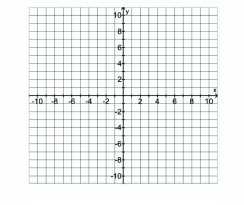
PRACTICE SET (Skill Building)

@Quadratic Functions

1. Sketch the following graphs.

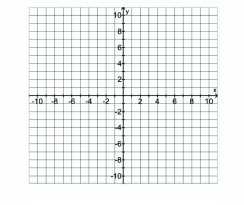
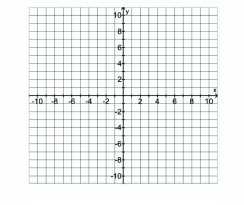
i. ��(��) = (�� − 3)! + 2

iv. �� = 2(�� + 1)!



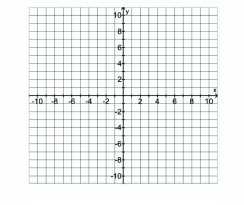
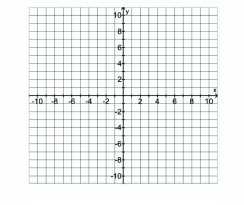
ii. �� = (�� − 1)! − 2

v. �� = −2(�� − 2)! + 2



iii. �� = −(�� − 1)! + 4

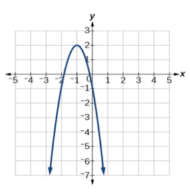
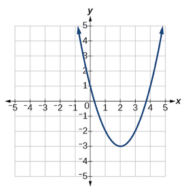
vi. �� = − "! (�� + 2)! − 4

2

2. For the following questions, write the equation for the graphed function.

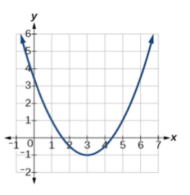
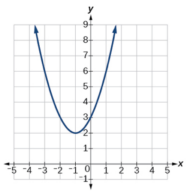
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4.



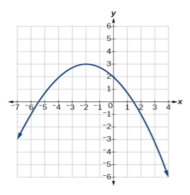
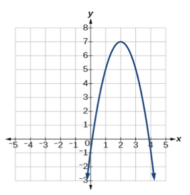
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5.



3.

6.

3

3. Convert given equation of quadratic functions into other two forms.

| Standard Form | Factored Form | Vertex Form |
| --- | --- | --- |
|  |  | �� = (�� − 3)! − 36 |
| �� = ��! + 10�� + 24 |  |  |
|  | �� = 4��(�� − 1) |  |
| �� = ��! − 4�� − 21 |  |  |

4

| �� = 3��! + 3�� − 18 |  |  |
| --- | --- | --- |
|  | �� = −3(�� − 5)(�� − 1) |  |
|  |  | �� = −2(�� − 1)! + 8 |
| �� = −��! − 2�� |  |  |

5

4. Fill in the blanks for the following questions. i. ��! − 4�� + \_\_\_ = (�� − \_\_\_ )!

ii. ��! + 10�� + \_\_\_ = (�� + \_\_\_ )!

iii. ��! − 18�� + \_\_\_ = (�� − \_\_\_ )!

iv. ��! + 20�� + \_\_\_ = (�� + \_\_\_ )!

v. ��! − 8�� + \_\_\_ = (�� − \_\_\_ )!

vi. ��! + 12�� + \_\_\_ = (�� + \_\_\_ )!

vii. ��! + 18�� + \_\_\_ = (�� + \_\_\_ )! viii.��! − \_\_\_ �� + 121 = (�� − \_\_\_ )! ix. ��! − 24�� + \_\_\_ = (�� − \_\_\_ )! x. ��! − \_\_\_ �� + 100 = (�� + \_\_\_ )! xi. ��! − \_\_\_ �� + \_\_\_ = (�� + 5)! xii. ��! − \_\_\_ �� + \_\_\_ = (�� − 4)!

6

5. Find the vertex form for the following quadratic function given in standard form by completing the square method. State if the vertex is a maximum or minimum point.

i. �� = ��! + 10�� + 15 ii. �� = ��! + 14�� − 14 iii. �� = 2��! + 12�� + 17 iv. �� = −��! + 4�� − 10

v. �� = ��! − 8�� − 4 vi. �� = −��! + 2�� + 4

vii. �� = 2��! − 12�� + 22 viii.�� = 4��! + 64�� + 156

7

6. Evaluate. i. ��"#

ii. ��"$

iii. ��%#

iv. ��&'

v. ��(' vi. ")!"vii. ")#viii. ")!$

7. Solve each of the following equation. <<Complete square method may be used>> i. ��! + 36 = 0 ii. ��! = −25

iii. ��! + 2�� + 5 = 0 iv. 2��! + 2�� + 5 = 0

v. ��! = −8 vi. ��! + 27 = 0 vii. ��! − 4�� + 5 = 0 viii.��! + 8�� + 6 = 0 ix. ��! − 4�� + 1 = 0 x. ��(�� − 2) = 1

8

8. Write a quadratic function in the vertex form whose graph has the given vertex and passes through the given point.

7. Vertex: (2,3) Point: (0,7) 8. Vertex: (-1,4) Point: (1,8)

Write a quadratic function in intercept form whose graph has the given ��-intercepts and passes through the given point.

9. ��-intercepts: 2,4 Point: (1,3) 10. ��-intercepts: 3,5 Point: (2,3)

9

9. Attempt each of the following questions.

i. A quadratic function with rational coefficients has a root at �� = 5 + √3. a) What is the other root? Also, write down the equation for the axis of symmetry.

b) Does function cross the X-axis? Justify.

c) Find all possible equations of the above quadratic function.

d) Find an equation of the above quadratic function that passes through (-3,4)

ii. A quadratic function with real coefficients has a root at �� = 2 + ��. a) What is the other root? Also, write down the equation for the axis of symmetry.

b) Does function cross the X-axis? Justify.

c) Find all possible equation of the above quadratic function.

d) Find an equation of the above quadratic function that passes through (-2,4)

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iii. A quadratic function with real coefficients has a root at �� = 2��.

a) What is the other root? Also write down the equation for the axis of symmetry. b) Does function cross the X-axis? Justify.

c) Find all possible equation of the above quadratic function.

d) Find an equation of the above quadratic function that passes through (0,3).

10. Attempt each of the following questions.

i. ��(��) = ����! − 8�� − 3 has a relative maximum at �� = −2. Find ��.

ii. ��(��) = ����! + 6�� − 2 has relative minimum at �� = −1. Find ��.

iii. ��(��) = 3��! + ���� + 8 has a minimum value of 2. Find ��.

iv. ��(��) = −��! + 2�� − �� has a maximum of −4. Find ��.

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11. Describe the transformation of ��(��) = ��! onto ��(��).

a) ��(��) = l"!��m! b) ��(��) = 3(�� − 1)!

c) ��(��) = −(�� + 3)! + 2

12. Attempt each of the following questions.

i. ��(��) = 2��! + 6�� maps onto ��(��) = −2��! − 6�� + 6. List all transformations in the proper order. ii. ��(��) = 2��! − 10�� + 8 maps onto ��(��) = ��! + 3��. List all transformations in the proper order.

iii. ��(��) = "+��! + 4 maps onto ��(��) = ��!. List all transformations in the proper order.

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@Quadratic Applications

13. Attempt each of the following questions.

i. A rectangular piece of cardboard measuring 40 in. by 30 in. is to be made into an open box with a base (bottom) of 900 ����! by cutting equal squares form the four corners and then bending up the sides. Find, to the nearest tenth of inch, the length of the side of the square that must be cut from each corner.

ii. The local park has a rectangular flower bed that measure 10 feet by 15 feet. The caretaker plans on doubling its area by adding a strip of uniform width around the flower bed. Determine the width of the strip.

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iii. The Lazy Boys are planning to have a concert during the Thanksgiving weekend. If the ticket is set to be $75 each, then 1800 tickets will be sold. For each $1 increase in the ticket price, 20 fewer tickets are sold. What should be the price of each ticket to maximum revenue?

iv. The current price of an amateur theater tickets is $20, and the venue typically sells 500 tickets. A survey found that for each $1 increase in ticket price, 10 fewer tickets are sold. What price per ticket will maximum the revenue? (Assume price of ticket as ‘x’ dollar)

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@Polynomial Graphing

14. Sketch the graph for each of the following polynomial function.

i. ��(��) = ��(�� − 2)(�� + 2) ii. ��(��) = (2 − ��)(�� + 2)+ iii. ��(��) = 4��!(�� − 2)! iv. ��(��) = ��+(�� − 1)+(�� + 2)

v. ��(��) = (3�� + 2)$(��! − 10�� + 25) vi. ��(��) = ��(4��! − 12�� + 9)(��! + 8�� + 16)

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vii. ��(��) = (2�� + 1)+(9��! − 6�� + 1) viii.��(��) = −��!(�� − 4)

ix. ��(��) = (�� − 1)(�� − 3)(�� + 2) x. ��(��) = (2 − ��)(�� + 2)+

15. Solve each of the following inequality.

xi. ��(��) = ��!(�� + 1)(2�� + 3) xii. ��(��) = (�� + 2)+(�� − 3)! xiii.��(��) = (�� + 3)(�� − 3)+

i. ��! ≤ 4�� + 5 ii. ��+ + 3 ≤ 3��! + �� iii. −2��+ + 4��! − 2�� ≤ −4 iv. ��' − 7 ≥ −6��!

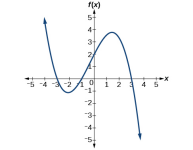
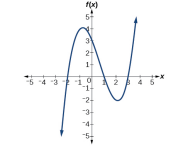
v. ��! > 2�� − 1

vi. 9�� + 9 ≤ ��+ + ��!vii. 5��+ − 3��! < 10�� − 6 viii.��' + 35��! − 36 ≥ 0

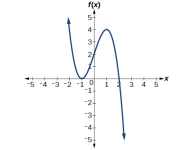
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16. Write an equation for the following graphs.

i. ii.



iii. iv.

17

vi.

v. 



vii. viii.



ix. x.



18

17. For the following exercises, use the given information about the polynomial graph to write the equation.

i. Degree 3; Zeros at �� = −2, �� = 1, and �� = 3; ��-intercept at (0, −4).

iii. Degree 4; Double root at �� = 4, and single roots at �� = 1 and �� = −2; ��-intercept at (0, −3).

ii. Degree 3; Zeros at �� = −3, �� = −2, and �� = 1; Passes through point (2, -20).

iv. Degree 5; Double root at �� = 1, and triple root at �� = 3; Passes through the point (2,15).

v. Degree 3; Zeros at = −5, �� = −2, and �� = 1; ��-intercept at (0,6).

vi. Degree 5; Double roots at �� = 3 and �� = 1, and single root at �� = −3; ��-intercept at (0,9) vii. Degree 3; Zeros at �� = 4, �� = 3, and �� = 2; ��-intercept at (0,-24).

viii.Degree 5; Double roots at �� = −3 and �� = 2 and single root at �� = −2; ��-intercept at (0,4). ix. Degree 4; Double roots at �� = "!and single roots at �� = 6 and �� − 2; Passes through point (1, 30). x. Degree 5; Double root at �� = −3 and triple zero at �� = 0. Passes through the point (1,32).

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@Long Division

18. For the following exercises, use long division to divide. Specify the quotient and the remainder. i. (��! + 5�� − 1) ÷ (�� − 1) ii. (2��! − 9�� − 5) ÷ (�� − 5)

iii. (��' + 4��+ − 6�� − 6) ÷ (��! − 3) iv. (4��! − 10�� + 6) ÷ (4�� + 2) v. (6��! − 25�� − 25) ÷ (6�� + 5) vi. (��' + 3��! − 6�� + 5) ÷ (�� − 2)

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@Remainder and Factor Theorem

19. For the following exercises, use the Remainder Theorem to find the remainder.

i. (��' − 9��! + 14) ÷ (�� − 2) ii. (3��+ − 2��! + �� − 4) ÷ (�� + 3)

iii. (6��! − 25�� − 25) ÷ (6�� + 5)

iv. (��' + 5��+ − 4�� − 17) ÷ (�� + 1) v. (−3��! + 6�� + 24) ÷ (�� − 4)

20. For the following exercises, use the Factor Theorem to verify given term as a factor of ��(��). If yes, then find all complex roots for the given polynomial function.

i. ��(��) = 2��+ − 9��! + 13�� − 6; �� − 1 ii. ��(��) = 2��+ + ��! − 5�� + 1; �� + 2

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iii. ��(��) = 3��+ + ��! − 20�� + 12; �� + 3 iv. ��(��) = 2��+ + 3��! + �� + 6; �� +

21. Attempt each of the following question.

i. Find the value of �� if the remainder of ��+ + ����! − �� + 2 when divided by (�� + 2) is 20.

ii. When 2��+ + ����! + ���� + 1 is divided by (�� − 1) and (�� − 2), the remainders are 4 and 15 respectively. Find the remainder when it is divided by (�� + 1).

iii. The expression ����+ − 8��! + ���� + 6 has a factor (��! − 2�� − 3). Calculate the value of �� and �� and hence find the other factor.

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22. For the following exercises, determine the least possible degree of the polynomial function shown.

i.

v. 



ii.

vi.



iii.

vii. 



iv.

viii.

23

ix.



x.



xi.



xii.



xiii.



xiv.



xv.

24

23. Attempt each of the following question.

i. A polynomial has all its real roots on the interval −3 < �� < 6. Three of them are double roots and exactly one is triple root. Use this information and the table of values below to determine the smallest possible degree for the polynomial. Justify your answer.

| �� | -3 | -1 | 0 | 2 | 3 | 5 | 6 |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ��(��) | 4 | -5 | 2 | 8 | -1 | -3 | 5 |

ii. A polynomial has all its real roots on the interval −7 < �� < 5. One of them is double root and exactly two of them are triple roots. Use this information and the table of values below to determine the smallest possible degree for the polynomial. Justify your answer.

| �� | -7 | -5 | -4 | -1 | 1 | 2 | 5 |
| --- | --- | --- | --- | --- | --- | --- | --- |
| ��(��) | -10 | -2 | 3 | 5 | -1 | 4 | 6 |

24. A polynomial function ��(��) has roots (zeroes/��-intercepts) at �� = 2, �� = 0 and �� = −2. If possible, then find the roots of the polynomial function ��(��). If not possible, explain why.

i. ��(��) = ��(�� − 1) ii. ��(��) = −��(��)

iii. ��(��) = −2��(3��) iv. ��(��) = 2�� l"'�� − 1m

25

vi. ��(��) = −4�� É"! (�� + 3)Ñ vii. ��(��) = �� l"!��m − 7

viii.��(��) = ��(�� + 2) ix. ��(��) = ��(2��) x. ��(��) = ��(−��)

xi. ��(��) = −��(�� + 3) xii. ��(��) = ��(��) − 3 xiii.��(��) = ��(�� − 1) − 2

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Question Set (Exam-style Questions)

1. Determine the type of complex roots of the quadratic function �� = ��! − 6�� + 30 without actual factorization or use of quadratic formula. Find the exact complex roots of the above quadratic function.

2. ��(��) = ��! maps onto ��(��) = 2��! − 8�� + 9. Find the transformations require to do so. 3. ��(��) = 2��! + 6�� − 9 maps onto ��(��) = −4��! − 4�� + 1. List all transformations in the proper order.

4. ℎ(��) = ����! + 6�� + 9 has a relative minimum at �� = −2, find ‘a’.

5. Find the value of ‘b’ given that ��(��) = 3��! + ���� + 13 has a minimum of 4.

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6. A quadratic function has a root at �� = −2 − 3�� .

a. What is the other root? Also write down the equation of axis of symmetry.

b. Find all possible equations of the above quadratic function.

c. Find the equation if the above quadratic function passes through (-1,10). 

d. Sketch the graph.

7. A quadratic function with rational coefficients has a root at �� = 6 + √3.

a. Write down its other root. Hence, find the equation of the axis of symmetry of the quadratic function. b. Find an equation for all possible quadratic functions with a root of �� = 6 + √3 .

c. Find an equation for above quadratic function if it passes through (1, 46).

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8. A quadratic function with rational coefficients has a root at �� = 3 + √5 .

a. What is the other root?

b. Write down the equation of its axis of symmetry.

c. Create an equation that represents all possible quadratic functions with rational coefficients with a root at �� = 3 + √5

d. Given that the function passes through (7, 12), Calculate the exact value of its leading coefficient.

9. You own a small airplane that holds a maximum of 40 passengers. If 10 passengers take the flight, then the charge per passenger is $35 per flight. The charge is reduced by $1.50 per passenger for each additional passenger. What number of passengers on a flight will maximize the revenue?

10. If x chairs are made per week, each one will cost (−50 + '##

, ) dollars, and the total receipts per week for

selling them would be (550 + 2��!) dollars. How many chairs should be made per week in order to maximize profits?

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11. The total cost of producing x toasters per day is given by ��(��) = "$��! + 30�� + 25dollars, and the selling price of each toaster is (54 − ��) dollars. How many toasters should be produced each day in order to maximize the total profit?

12. The total cost of producing �� blankets per day is ��(��) = "

',~~"~~ + 8�� + 20, in dollars, and each blanket may be

sold for l23 − "!��m dollars. How many blankets should be produced per day to maximize the total profit?

13. After conducting some market research, a bubble tea company have found that if they price their tea at $4, they will sell 360 teas a day. Every $2 increase in price, the number of tea sales is reduced by 30 each day. What is the maximum possible daily revenue?

14. A fruit grower has 400 crates of grapes ready for market and will have 20 more crates each day that shipment is delayed. The present price is $60 per crate, however, for each shipment is delayed, the price per crate decreases by $2. How many days the grower should delay shipment in order to maximize revenue?

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15. Tickets are about to go on sale for the latest high school play. After conducting some market research, students have found that if they price the tickets at $10, they will fill 1200 seats. For each dollar increase in price, they will fill 50 fewer seats.

a. Write a function that represents the expected attendance, ��(��) of this event as a function of ticket price xb. What price should they sell the tickets for if they want to maximize revenue? Justify your answer.

16. The manufacturer has been selling light bulbs for $2 each. People have been buying 100 light bulbs/month. If the manufacturer increases the price by $1, they will sell 4 fewer bulbs.

a. Write a function that represents the expected number of bulbs sold, ��(��), as a function of price, ��.b. What price should they sell the bulbs for if they want to maximizerevenue? Justify your answer.

17. For his catalyst project, Max offers to conduct some market research for the booster booth so they can optimize their pricing. Currently, when priced at $4, the booth sells 240 A4 notebooks a day. He discovers that if they increase the price by $2, they sell 60 less notebooks each day.

a. Write a function that represents the expected number of notebooks sold, ��(��), as a function of price, ��. b. What price should they sell the notebooks for if they want to maximize revenue? Justify your answer.

31

18. A farmer has 850 feet of fencing to construct a rectangular pen with 8 equal sized stalls, as shown in the given figure. Find the dimensions of the pen that has the largest area.

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |

19. A farmer has 80 meters of fence to build a pen for his animals. One side of the pen will be his house. What dimensions will produce the maximum area?

20. Given that ��(��) = "!��! + (�� + 1)�� + 3��! − 3, find the range of values of �� for which ��(��) > 0 for all real values of x.

21. Prove that the equation 2x! + 2bx + b − 36 = 0 has two distinct real roots for all values �� ∈ ℝ.

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22. Consider the polynomial Q(x) = 3x! + 3px + p −2 where �� �� ℝ

a. Prove that this polynomial will always have two distinct (different) real roots.

b. What value(s) of �� will minimize the distance between the roots?

23. Match each graph with the equations shown below. If there is no match, write “none of these”

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24. Find the equation for the function shown below. You may leave your answer in factored form. a) b) 

c) d) 

e) 

25. Which of the following could be a graph for the function, Q(x) = ax(2 − x)(x + 3)+ if �� < 0. Justify.34

26. A quadratic function has a root of �� = 3 + ��. Circle each statement that is true and provide a justification. I. The function could cross the x-axis

II. �� = 3 is the axis of symmetry

III. f(x) = a(x! − 6x + 10) is all possible equations with �� ∈ ℝ, �� ≠ 0

I. Justification: II. Justification: III. Justification:

27. A quadratic function has a root of �� = 3 + ��. Circle each statement that could be true and provide a justification.

I. The function doesn’t cross the x-axis.

II. The axis of symmetry cannot be determined

III. III. ��(��) = ��! − 6�� + 10

Justification: Justification: Justification:

28. Below is a graph of a polynomial function �� = ��(��).

a. Describe the end behavior of �� = ��(��).

b. List the real zeroes of p(x) along with their respective multiplicities. 

c. List the local minimums and local maximums of the graph of �� = ��(��).

d. What can be said about the degree and leading coefficient of �� = ��(��).

e. It turns out that �� = ��(��) is a seventh-degree polynomial. How can this be?

29. Sketch the following.

a. ��(��) = −3��!(�� + 2)+(�� − 4) b. ��(��) = ��' − 10��! + 9

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30. Sketch ��(��) = 2��- − 40��( + 200��'. Also, describe the end behavior of ��(��). 31. Sketch ��(��) = ��& − 14��$ + 49��+ and describe the end behavior of the function.

32. Sketch the graph of a polynomial function with all of the following characteristics. a. Degree 5 with positive leading coefficient.

b. Exactly one triple root.

c. ��(2) = 3 and ��(4) = −2.

d. Goes through the points (0,5) and (3,0).

33. Sketch a function with the following characteristics.

a. Degree 6 with a negative leading coefficient.

b. No real roots.

c. ��(0) = −8.

d. It has the maximum number of turning points for a degree 6 polynomial.

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34. A polynomial function ��(��) has roots (zeroes/ x intercepts) of �� = 2, �� = 0 and �� = −2. If possible, find the roots of the polynomial function g(x). If not possible, explain why.

a. ��(��) = 4��(��) – 3 b. ��(��) = −4��(1 − ,!)

35. A quantic (deg 5) polynomial function g(x) has single root at x=-2 and double roots at x=3 and 7. If possible then find the roots of the given polynomial functions.

a. ��(��) = −3��(2�� + 10) b. ℎ(��) = −��(−��) + 1

36. A polynomial ��(��) has roots at �� = 9, �� = −3 ������ �� = −7. if possible, find the roots of the polynomial �� = −��(4 − "!��)

37. A polynomial has all of its roots on the interval −2 < �� < 6. All of its roots are real. Three of them are double roots and two of them are triple roots. Use that information and the table of values below to determine the smallest possible degree for this polynomial. Justify your answer.



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38. A polynomial has all of its roots on the interval (-6,6). A table of values is shown below. It is known that the polynomial contains three double roots and one triple root. What is the minimum degree for the polynomial? Justify your answer.



39. A polynomial has all of its roots on the interval (−3,3). A table of values is shown below. It is known that the polynomial contains three double roots. What is the minimum possible degree for the polynomial? Justify your answer.



40. ��(��) = 2��$ + 14��' − 12��+ − 84��! + 10�� + 70 and (�� + √5) is a factor.

a. Find its completely factored form.

b. Find all complex roots of ��(��).

c. Classify the roots according to their type: (Imaginary, real and rational, real and irrational).

41. ��(��) = ��$ − 2��' − 7��+ + 10��! + 10�� and ��(√5)= 0.

a. Find its completely factored form.

b. List all complex roots of ��(��).

c. Classify the roots according to their type: (Imaginary, real but not irrational, real and irrational)

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42. ��(��) leaves no remainder when divided by �� −√3. Factor ��(��) = ��' − 2��+ − 6��! + 6�� + 9 completely. Hence, list all complex roots of f(x).

43. (x+ − k!x! + 3kx − 4) is divided by (�� − 2) and the remainder is zero. Find ‘k’. 44. The remainder is zero when x+ + x!a! + .(x – 237 is divided by (�� − 6), find ‘a’.

39

45. True or False: if False, explain why or provide a counterexample.

a. A polynomial of the 5th degree can have 2 real roots and 3 imaginary roots. b. A polynomial function of degree 8 can have exactly 5 real roots.

c. A polynomial function of degree 7 must have at least one rational root. d. A 44th-degree polynomial function can have exactly 12 relative extrema (Turning Points).

e. Every even degree polynomial function is an even function.

f. A polynomial function which is an odd function is also of an odd degree. g. An odd-degree polynomial has a range of all real numbers.

h. An even degree polynomial has a domain of all real numbers.

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Answer Scheme Practice Set

1. i. 

ii. iii. iv. v. 

vi. 

2. i. ��(��) = (�� − 2)! − 3

ii. ��(��) = (�� + 1)! + 2

iii. ��(��) = −2(�� − 2)! + 7

iv. ��(��) = −3(�� + 1)! + 2

v. ��(��) = "! (�� − 3)! − 2

vi. ��(��) = − "' (�� + 2)! + 3

3. i. Standard: �� = ��! − 6�� − 27 Factored: �� = (�� − 9)(�� + 3)

ii. Factored: �� = (�� + 6)(�� + 4) Vertex: �� = (�� + 5)! − 1

iii. Standard: �� = 4��! − 4��

Vertex: �� = 4 l�� − "!m!− 1

iv. Factored: �� = (�� − 7)(�� + 3) Vertex: �� = (�� − 2)! − 25

v. Factored: �� = 3(�� − 2)(�� + 3) Vertex: �� = 3 l�� + "!m!− &$'

vi. Standard: �� = −3��! + 18�� − 15 Vertex: �� = −3(�� − 3)! − 12

vii. Standard: �� = −2��! + 4�� − 6 Factored: �� = −2(�� − 3)(�� + 1) viii. Factored: �� = −��(�� + 2)

Vertex: �� = −(�� + 1)! + 1

4. i. 4,2

ii. 25,5

iii. 81,9

41

iv. 100,10

v. 16,4

vi. 36,6

vii.81,9

viii. 22,11

ix. 144,12

x. 20,10

xi. 10,25

xii. 8,16

5. i. (��+5)!− 10, ������ ii. (��+7)!− 63, ������

iii. (2(��+3)!−1, ������ iv. −(��−2)!−6, ������ v. (��−4)!− 20, ������

vi. −(��−1)!+5, ������ vii. 2(��−3)!+4, ������ viii. 4(��+8)!− 100, ������

6. i. -1

ii. -i

iii. -1

iv. -1

v. 1

vi. 1

vii. i

viii. i

7. i. ±6��

ii. ±5��

iii. −1±2��

iv. −"!±+!��

v. ±2√2��

vi. ±3√3��

vii. 2±��

viii. −4±√10

ix. 2±√3

x. 1±√2

8. i. ��=(��−2)!+3

ii. ��=(��+1)!+4

iii. ��=(��−2)(��−4)

iv. ��=(��−3)(��−5)

9. i. a. ��=5−√3

b. Yes

c. ��(��)=��(��!− 10��+ 22)

d. ��(��)='("(��!− 10��+ 22)

ii. a. ��=2−��

b. No

c. ��(��)=��(��!−4��+5)

d. ��(��)='"&(��!−4��+5)

iii. a. ��=−2��

b. No

c. ��(��)=��(��!+4)

d. ��(��)=+'(��!+4)

10. i. -2

ii. 3

iii. ±√72

iv. 5

11. a. HD by 2

b. VD by 3, right by 1

c. Reflection over x-axis, left by 3, up by 2

12. i. Reflection over x-axis, up by 6 ii. VD by "!, left by 1, down by %!

iii. VD by 3, down by 12

13. i. 2.29 inches

ii. 2.5 ft

iii. $82.5

iv. $35

42

14. i. ii. iii. iv. 

v. vi. vii. viii. 

ix. x. xi. xii. xiii. 

15. i. [−1,5]

ii. (−∞, −1] ∪ [1,3]

iii. [2, ∞]

iv. (−∞, −1] ∪ [1, ∞)

v. (−∞, 1) ∪ (1, ∞)

vi. [−3, −1] ∪ [3, ∞)

vii. ù−∞, −√2û ∪ l+$ , √2m

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viii. (−∞,−1]∪[1,∞)

16. i. "!(��+2)(��−1)(��−3)

ii. −!%(��+3)(��+1)(��−3)

iii. 2(��+1)!(��−2)

iv. "'(��+2)!(��−3)

v. −"!'(��+3)(��+2)(��−3)(��−4) vi. −"-(��+4)(��+2)(��−1)(��−3) vii. ""!(��+2)!(��−3)!

ix. "((��+3)(��+2)(��−1)+

x. ����+(��+3)(��+1)(��−2)!(��−3)

17. i. −!+(��+2)(��−1)(��−3) ii. −(��+3)(��+2)(��−1)

iii. ++!(��−4)!(��−1)(��+2)

iv.−15(��−1)!(��−3)+

v. −+$(��+5)(��+2)(��−1)

vi. −"+(��−3)!(��−1)(��+3)

vii.(��−4)(��−3)(��−2)

viii. −"%(��+3)!(��−2)(��+2) ix. 24l��−"!m!(��−6)(��−2)

��.2��+(��+3)!

18. i. Quotient: x+6

Remainder:5

ii. Quotient: 2x+1

Remainder:0

iii. Quotient:��!+4��+3

Remainder: 6x+3

iv. Quotient:x-3

Remainder: 12

v. Quotient:x-5

Remainder:0

vi. Quotient:��++2��!+7��+8 Remainder: 21

19. i. -6

ii. -106

20. i. Yes, ��=1,+!,2

ii. No

iii. Yes, ��=−3,!+,2

iv. Yes, ��=−2,−"'±√!+'��

21. i. k=6

ii. a=-2, b=3

iii. a=3, b=-5

22. i. 3

ii. 2

iii. 5

iv. 1

v. 3

vi. 4

vii. 5

viii. 2

23. i. 12

ii. 9

24. i. ��=3,1,−1

ii. ��=2,0,−2

iii. ��=!+,0,−!+

iv. ��= 12,4,−4

vi. ��=1,−3,−7

vii. Not possible

viii. ��=0,−2,−4

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ix. �� = 1,0, −1 x. �� = −2,0,2

xi. �� = −1, −3, −5

xii. Not possible xiii. Not possible

Question Set

1. �� = 3 ± √21��

2. Vertical dilation SF 2, Translate 2 units right, 1 unit up 3. Reflect on ��-axis

Vertical Dilation SF 2

Translate 1 unit left

Translate +"! unit up

4. +!

5. ±6√2

6. a. �� = −2 + 3��, axis of symmetry: �� = −2 b. �� = ��(��! + 4�� + 13), �� ≠ 0

c. �� = $%

d. 

7. a. �� = 6 − √3, axis of symmetry : �� = 6

b. �� = ��(��! − 12�� + 33), �� ≠ 0

c. �� = !+"" (��! − 12�� + 33)

8. a. �� = 3 − √5

b. �� = 3

c. �� = ��(��! − 6�� + 4), �� ≠ 0

d. �� = "!""

9. 13

45

10. 12 or 13

11. 10

12. 10

13. 2940

14. 5

15. a. ��(��) = −50�� + 1700

b. 17

16. a. ��(��) = −4�� + 108

b. 13

17. a. ��(��) = −30�� + 120

b. 2

18. 141.6 x 42.52

19. 40 x 20

20. �� < −1, �� > &$

21. Proven

22. a. Proven

b. �� = !+

23. E F D

B C A

24. a. �� = − +' (�� + 2)(�� − 1)!(�� − 3)

b. �� = − "( (�� + 3)!(�� − 1)+(�� − 4)

c. �� = ""( (�� + 4)!(�� + 2)(�� − 1)+

d. �� = −2(�� + 2)(�� − 3)!

e. �� = − "! (�� + 3)(�� + 2)(�� − 1)!

25. ��

26. II and III

27. I

28. a. as �� → ∞, ��(��) = −∞, as �� → −∞, ��(��) = −∞ b. �� = −1.5, �� = 0, �� = 1

c. Minimum (−0.773, −2.88), Maximum (0.32, 0.532) 29. a. 

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b. 

30. 

31. 

32. 33. 

34. a. Not possible b. �� = −2, 0 , 6

35. a. �� = −6, − &! ,+! b. Not possible

36. �� = 22,14, −10 37. 13

47

38. 13

39. 10

40. a. 2(�� + 1)(�� − 1)(�� + 7)(��! − 5) b. �� = −7, ±1, ±√5

c. Rational : �� = −7, ±1, Irrational : �� = ±√5 41. a. ��(��) = ��(��! − 5)(��! − 2�� − 2) b. �� = 0, ±√5, 1 ± √3

c. Rational : �� = 0, Irrational : �� = ±5,1 ± √3 42. a. ��(��) = (��! − 3)(�� − 3)(�� + 1) b. �� = ±√3, 3, −1

c. Rational : �� = 3, −1, Irrational : �� = ±√3

43. �� = − "! , �� = 2

44. �� = +& , − &%

45. a. False

b. False

c. False

d. False

e. False

f. True

g. True

h. True

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